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PRV Stability Inlet Line Critical Length

A Short Communication

authored by

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1 Introduction

Research by Chiyoda [1], Pentair [2] and ioMosaic [3] showed that pressure relief valve (PRV) instability leading to flutter and/or chatter is due to the coupling of the PRV disk motion with the quarter wave pipe/fluid mode frequency without resonance. Izuchi [1] simplified his detailed modeling analysis to restrict the inlet line length for stable PRV operation and derived an analytical expression for simple inlet line geometries.

In a recent presentation (March 10, 2021) on PRV stability by the Center of Safety Excellence (CSE) [4], Keszthelyi et al. validated, using actual measurements, the inlet line critical length criteria proposed by several authors. The measurements were obtained at CSE's PRV test facility in Germany. The evaluation showed that the inlet line critical length criterion originally derived by Izuchi [1] and later simplified by Melhem [5] showed the best performance vs. actual measurements of critical line length.

In his presentation, Keszthelyi [4] suggested that Izuchi's original derivation of inlet line critical length is missing 1 from a square root of the critical line length, L_{crit}, equation:

$$L_{crit} = \frac{\pi c}{2\omega_n\sqrt{\frac{x+x_o}{x}+1}} = \frac{c}{4f_n\sqrt{\frac{x+x_o}{x}+1}} \quad \text{and} \quad f_n = \frac{\omega_n}{2\pi} \tag{1}$$

Where c is the speed of sound, ω_n is the undamped circular natural frequency, x_o is the initial compression of the PRV spring at zero lift, f_n is the PRV frequency, x is the PRV disk lift, and L_{crit} is the critical inlet line length.

This short communication confirms that the original equation is correct (1 is not needed) and was developed for initial opening of the PRV. The original equation is recommended for use in PRV stability screening because it was derived without the impact of damping. However, 1 should be added to the square root term at full opening of the pressure relief valve [6].

2 Original Derivation by Izuchi

Izuchi derived the inlet line critical length criteria based on zero damping as shown in his 2010 paper [1]:

$$L_{crit} = \frac{\pi c}{2\omega_n\sqrt{\underbrace{\frac{x+x_o}{x}}_{K_1}+\left[1-\beta\frac{df}{dZ}\right]}} \tag{2}$$

$$\beta = \frac{PA_H}{K_s} \tag{3}$$

The term $\left[1-\beta\frac{df}{dZ}\right]$ is PRV specific. Z is the spring displacement, P is the average pressure at the PRV inlet, A_H is the area of the disk holder, f is the lift force function, and K_s is the

Table 1: PRV data for 1E2 from [7], page 42

$\overline{Z}$	0.0006	0.001	0.002	0.003	(m)
K_1	10.8	6.9	3.9	3.0	
$\overline{P}$	1.90E+06	1.95E+06	2.10E+06	2.30E+06	(Pa)
df/dZ	180	100	50	25	(1/m)
β	5.56E-03	5.71E-03	6.15E-03	6.73E-03	(m)
$\overline{W}$	0.16	0.28	0.60	0.98	(kg/s)
μ	19.5	11.7	5.9	3.9	
$1-\beta\, df/dz$	0	0.43	0.69	0.83	

spring constant. For a small PRV lift, $\left[1-\beta\frac{df}{dZ}\right] \simeq 0$ and for full PRV lift $\left[1-\beta\frac{df}{dZ}\right] \simeq 1$. This is shown in Table 1 by Izuchi [7]. Table 2 illustrates the impact of initial and full PRV lift on the value of inlet line critical length [6] for the same PRV. As shown in Table 2, the suggestion to use the conditions at full lift yields a slightly more conservative answer. However, all three critical line length estimates assume zero damping. In real installations damping will always be present. Therefore, it is more appropriate as initially suggested by Izuchi [1], to use the conditions at initial lift.

3 Melhem Simplification

Melhem [5] simplified and converted the expression derived by Izuchi [1] to use maximum PRV lift, x_{max}, instead of x_o.

Initial Opening Conditions

$$\alpha = \sqrt{\frac{x}{x+x_o}} \tag{4}$$

$$L \leq L_{crit} = \frac{c\alpha}{4f_n} = \frac{c}{4f_n}\sqrt{\frac{x}{x+x_o}} \simeq \frac{\alpha}{2}ct_{valve} \tag{5}$$

Full Opening Conditions

$$\alpha = \sqrt{\frac{x}{2x+x_o}} \tag{6}$$

$$L \leq L_{crit} = \frac{c\alpha}{4f_n} = \frac{c}{4f_n}\sqrt{\frac{x}{2x+x_o}} \simeq \frac{\alpha}{2}ct_{valve} \tag{7}$$

Table 2: PRV data for 1E2 at initial and full PRV lift [6]

Z	m	0.0006	0.001	0.002	0.003
K1	-	10.8	6.9	3.9	3
P	Pa	1.90E+06	1.95E+06	2.10E+06	2.30E+06
df/dZ	1/m	180	100	50	25
β	m	5.56E-03	5.71E-03	6.15E-03	6.73E-03
W	kg/s	0.16	0.28	0.6	0.98
1-βdf/dz	-	-0.001	0.429	0.693	0.832
v	-	3.286	2.707	2.143	1.957
φ	-	0.152	0.185	0.233	0.255
c	m/s	350	350	350	350
ωn	1/s	471.2	471.2	471.2	471.2
L exact	m	0.355	0.431	0.544	0.596
L Chiyoda	m	0.355	0.444	0.591	0.674
L CSE	m	0.340	0.415	0.527	0.583

where x_o is the initial compression of the PRV spring at zero lift in m, f_n is the PRV frequency in Hz, x is the PRV disk lift in m, L_{crit} is the critical inlet line length in m, α is a valve lift parameter, and c is the speed of sound in the fluid/pipe system in $\mathrm{m/s}$.

x_o can be calculated from the set point of the PRV, the mass in motion, and the PRV spring constant:

$$x_o = \frac{P_{set} A_N - m_D g}{K_s} \tag{8}$$

where P_{set} is the set point in gauge pressure units, A_N is the PRV nozzle area, m_D is the PRV mass in motion and K_s is the spring constant.

Grolmes [8] developed an empirical method for the estimation of spring constants (K_s) and weights in motion (m_D) based on actual measurements of several PRVs and associated components.

$$K_s = C_1 \left[\frac{P_{set} A_N}{x_{max}}\right] = \underbrace{C_2 C_3}_{C_1} \left[\frac{P_{set} A_N}{x_{max}}\right] = \underbrace{\left(\frac{P_{full flow}}{P_{set}}\right)}_{C_2} \underbrace{\left(\frac{A_{pop}}{A_N}\right)}_{C_3} \left[\frac{P_{set} A_N}{x_{max}}\right] \tag{9}$$

where C_1, C_2, and C_3 are dimensionless constants close to 1 in magnitude. He also provided an empirical formula for estimating the weight in motion:

$$m_D = \frac{M_{PRV}}{100} \left(1.8 + 0.022 M_{PRV}\right) = 0.018 M_{PRV} + 0.00022 M_{PRV}^2 \tag{10}$$

where M_{PRV} is the valve body weight in lb including a 150 # flange.

If we assume that m_D is small compared to $P_{set}A_N$, then we can approximate x_o by:

$$x_o = \frac{P_{set}A_N - m_D g}{\frac{P_{full}}{P_{set}} \frac{A_{pop}}{A_N} \frac{P_{set}A_N}{x_{max}}} \simeq \frac{P_{set}A_N}{\frac{P_{full}}{P_{set}} \frac{A_{pop}}{A_N} \frac{P_{set}A_N}{x_{max}}} \tag{11}$$

where x_{max} is the maximum PRV disk lift in m. Typically $\frac{P_{full}}{P_{set}}$ is 1.1 and $\frac{A_{pop}}{A_N}$ ranges from 1.2 to 1.3 (say 1.3), we can further simplify Izuchi's stability criterion to the following:

Initial Opening Conditions

$$\boxed{L \leq L_{crit} = \frac{c}{4f_n}\sqrt{\frac{1.43x}{1.43x + x_{max}}}} \tag{12}$$

If the disk is at maximum lift, then the inlet line length should be limited to:

$$L \leq L_{crit} = \frac{c}{4f_n}\sqrt{\frac{1.43x_{max}}{1.43x_{max} + x_{max}}} \leq 0.77\frac{c}{4f_n} \tag{13}$$

At 60 % of maximum disk lift, the inlet line length required for stable operation becomes:

$$\boxed{L \leq L_{crit} = 0.68\frac{c}{4f_n}} \tag{14}$$

Full Opening Conditions

$$\boxed{L \leq L_{crit} = \frac{c}{4f_n}\sqrt{\frac{1.43x}{2.86x + x_{max}}}} \tag{15}$$

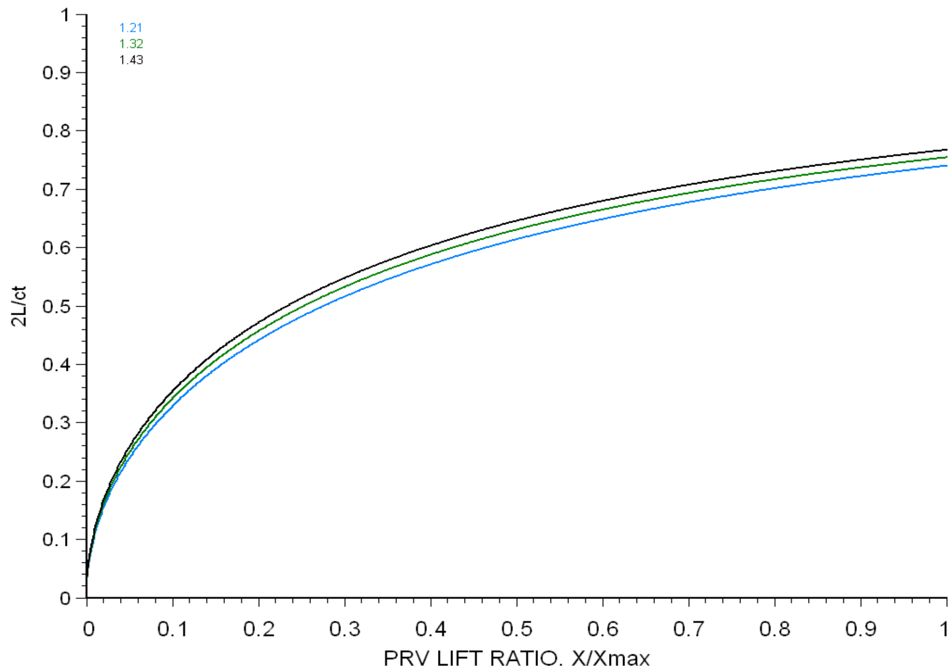
If the disk is at maximum lift, then the inlet line length should be limited to:

$$L \leq L_{crit} = \frac{c}{4f_n}\sqrt{\frac{1.43x_{max}}{2.86x_{max} + x_{max}}} \leq 0.61\frac{c}{4f_n} \tag{16}$$

At 60 % of maximum disk lift, the inlet line length required for stable operation becomes:

$$\boxed{L \leq L_{crit} = 0.56\frac{c}{4f_n}} \tag{17}$$

Figure 1: Inlet line length stability limit as a function of disk lift ratio



One can always solve for the actual PRV disk lift from a detailed force balance (see Melhem [3]) and calculate the required inlet line length for PRV stability in a more detailed manner. The expression above can be related to the PRV opening/closing time:

$$t_{valve} \simeq \frac{1}{2f_n} \tag{18}$$

$$\frac{2L}{c} \leq \underbrace{\left(\sqrt{\frac{1.43\frac{x}{x_{max}}}{1.43\frac{x}{x_{max}}+1}}\right)}_{\alpha} t_{valve} \tag{19}$$

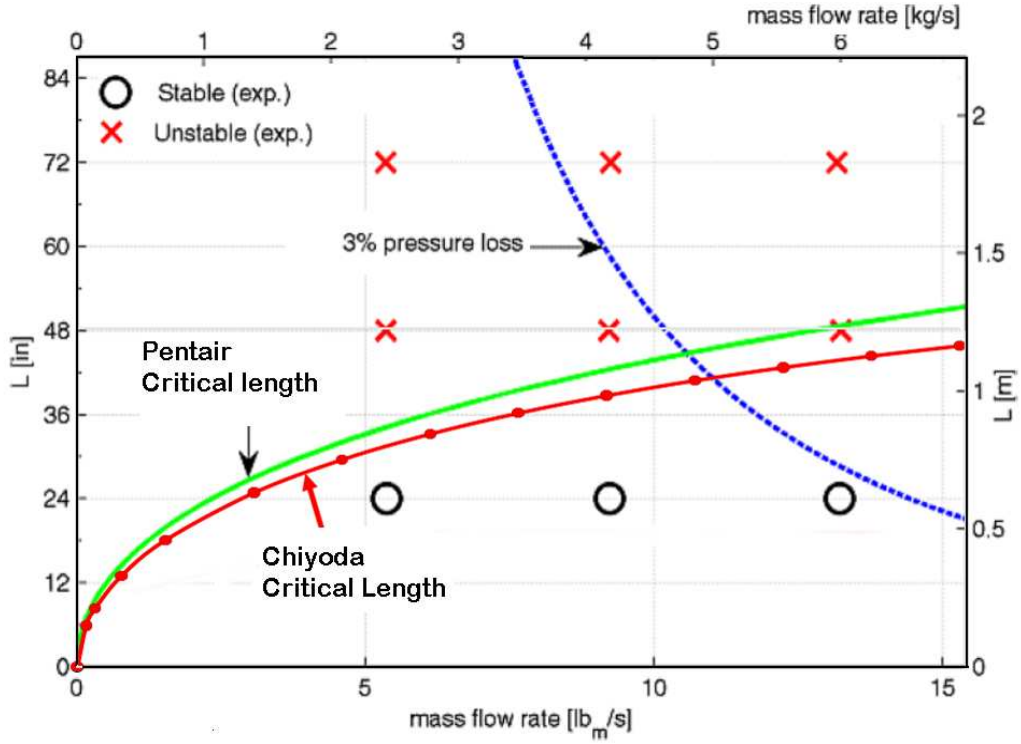
$$L \leq L_{crit} = \frac{\alpha}{2} c t_{valve} \tag{20}$$

where α typically ranges from 0.68 to 0.77 (for A_{pop}/A_N ranging from 1.2 to 1.3). This expression is approximately 30 % less than the acoustic length stability criteria provided in the recently published API-520 part II in Appendix C [9].

If we plot $\frac{2L}{ct_{valve}}$ vs. $\frac{x}{x_{max}}$ at three different ratios of A_{pop}/A_N we get the behavior shown in Figure 1. The critical length criteria proposed by Chiyoda (Izuchi) [1] is consistent with the critical length criteria proposed by Pentair [2]. This is illustrated in Figure 2. The Pentair measured experimental data are shown as open circles and X symbols [10]. The Pentair analytical critical length model is superimposed over the measured data in green. The red line represents the critical length criteria developed by Chiyoda (Izuchi) [1].

If we use the last data point from [3] (Table 8) to illustrate the quarter wave screening procedure

Figure 2: Inlet line length stability limit predictions for 2J3 PRV vs. measured data by Pentair



assuming a value of A_{pop}/A_N of 1.20, or $\alpha = 1.32$.

$$\frac{2L}{ct_{valve}} = \frac{2 \times 6 \times 0.3048}{352 \times 0.0319} = 0.33 \tag{21}$$

The critical disk lift ratio can be read from Figure 1 or calculated from:

$$\frac{x}{x_{max}} = \frac{0.33^2}{1.32\,(1 - 0.33^2)} = 0.0925 \tag{22}$$

As illustrated in Figure 2, the 3 % irreversible inlet pressure loss rule is not sufficient to guarantee PRV stability. The simplified API force balance [9] developed by Melhem [5] can be extended to include the simplified quarter wave stability criteria.

4 Additional Reading

ioMosaic has published extensively on the science of PRV stability. The following references are suggested additional reading [3, 5, 11, 12, 13]:

1. Analysis of PRV Stability in Relief Systems. Part I - Detailed Dynamics

2. Analysis of PRV Stability in Relief Systems. Part II - Screening
3. Analysis of PRV Stability in Relief Systems. Part III - How to Avoid the Singing Pressure Relief Valve Problem
4. Analysis of PRV Stability in Relief Systems. Part IV - On the Estimation of Speed of Sound and Thermodynamic Properties for Fluid Flow and PRV Stability
5. Analysis of PRV Stability in Relief Systems. Part V - Get a Handle on PRV Stability

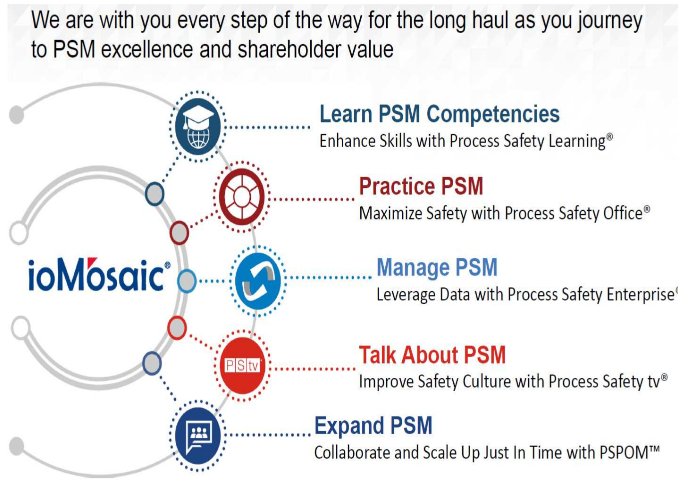
In particular, the Part V publication includes a video paper and several animations of PRV stability dynamics that illustrate the key concepts PRV stability.

5 Conclusions

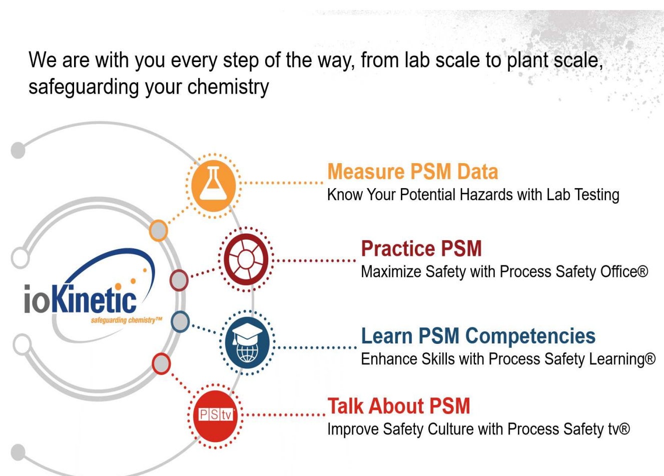
The original critical line length criterion as derived by Izuchi [1] for initial opening by the pressure relief valve and later simplified by Melhem [5] is recommended for use in pressure relief valve stability screening for simple inlet line geometries.

How can we help?

In addition to our deep experience in process safety management (PSM) and the conduct of large-scale site wide relief systems evaluations by both static and dynamic methods, we understand the many non-technical and subtle aspects of regulatory compliance and legal requirements. When you work with ioMosaic you have a trusted ISO certified partner that you can rely on for assistance and support with the lifecycle costs of relief systems to achieve optimal risk reduction and PSM compliance that you can evergreen. We invite you to connect the dots with ioMosaic.



We also offer laboratory testing services through ioKinetic for the characterization of chemical reactivity and dust/flammability hazards. ioKinetic is an ISO accredited, ultra-modern testing facility that can assist in minimizing operational risks. Our experienced professionals will help you define what you need, conduct the testing, interpret the data, and conduct detailed analysis. All with the goal of helping you identify your hazards, define and control your risk.



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References

[1] Hisao Izuchi. Stability analysis of safety valve. In *10th Topical Conference on Gas Utilization*. AIChE, AIChE, 2010.

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[13] G. A. Melhem. Analysis of PRV stability in relief systems. Part V - Get a handle on PRV stability. *ioMosaic Corporation White Paper*, 2019.

About the Author

Dr. Melhem is an internationally known pressure relief and flare systems, chemical reaction systems, process safety, and risk analysis expert. In this regard he has provided consulting, design services, expert testimony, incident investigation, and incident reconstruction for a large number of clients. Since 1988, he has conducted and participated in numerous studies focused on the risks associated with process industries fixed facilities, facility siting, business interruption, and transportation.

Prior to founding ioMosaic Corporation, Dr. Melhem was president of Pyxsys Corporation; a technology subsidiary of Arthur D. Little Inc. Prior to Pyxsys and during his twelve years tenure at Arthur D. Little, Dr. Melhem was a vice president of Arthur D. Little and managing director of its Global Safety and Risk Management Practice and Process Safety and Reaction Engineering Laboratories.

Dr. Melhem holds a Ph.D. and an M.S. in Chemical Engineering, as well as a B.S. in Chemical Engineering with a minor in Industrial Engineering, all from Northeastern University. In addition, he has completed executive training in the areas of Finance and Strategic Sales Management at the Harvard Business School. Dr. Melhem is a Fellow of the American Institute of Chemical Engineers (AIChE) and Vice Chair of the AIChE Design Institute for Emergency Relief Systems (DiERS).

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